New Marking Scale
S6/S7 Mathematics 3 and 5 Period

The exemplars below illustrate the procedures outlined in the Matrix Manual Mathematics, which accompanies this document. Please note that the approach is the same for both the 3P and 5P mathematics course. It is recommended that the manual is read prior to viewing these documents.

1. Generic Matrices

The original Excel spreadsheets for these matrices are available for use. Please refer to the communications received from the mathematics inspector.

1.1 Generic MA 3P Matrix

EUROPEAN BACCALAUREATE - Generic MA 3P Matrix

Element of Examination	Question	Learning Objective (specific syllabus reference(s))	Paper-specific Marking Scheme				
			Knowledge and Comprehension	Methods	Problem Solvng	Interpretation and Linking	Σ
Part A Non Calculator							
Analysis	A1						0,0
Analysis	A2						0,0
Analysis	A3						0,0
Analysis	A4						0,0
Analysis	A5						0,0
Probability	A6						0,0
Probability	A7						0,0
Statistics	A8						0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	12,0	18,0	8,0	2,0	40,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	

The fields marked in yellow can be filled in, all others are protected.		
The Sums are	green:	ok
	orange:	within tolerance
	red:	not allowed

For each individual question in the calculator paper (B1, B2 etc.), there is more flexibility in the spread of the marks but the overall weighting of the marks for the whole calculator paper must be respected.

Part B - Calculator							
B1	B1a						0,0
Analysis	B1b						0,0
Exactly 3 sub questions	B1c						0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	3,0	4,5	2,0	0,5	10,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	
B2	B2a						0,0
Analysis	B2b						0,0
	B2c						0,0
Minimum 4 sub questions	B2d						0,0
Maximum 5 sub questions	(B2e)						0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	4,5	6,8	3,0	0,8	15,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	
B3	B3a						0,0
Probability	B3b						0,0
	B3c						0,0
Minimum 4 sub questions	B3d						0,0
Maximum 5 sub questions	(B3e)						0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	4,5	6,8	3,0	0,8	15,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	
B4 (and B5)	B4a						0,0
Statistics	B4b						0,0
	B4c						0,0
If there is no B5, B4 must have...	B4d or B5a						0,0
Minimum 5 sub questions	B4e or B5b						0,0
Maximum 6 sub questions	B4f or B5c						0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	6,0	9,0	4,0	1,0	20,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	

Total Calculator						
	S	0,0	0,0	0,0	0,0	0,0
	%	0,0	0,0	0,0	0,0	
	Guideline:	18,0	27,0	12,0	3,0	60,0
	%	30,0	45,0	20,0	5,0	
	Tolerance (Points):	3,0	4,0	2,0	2,0	

Total A and B						
	S	0,0	0,0	0,0	0,0	0,0
	%	0,0	0,0	0,0	0,0	
	Guideline:	30,0	45,0	20,0	5,0	**100,0**
	Tolerance (Points):	4,0	5,0	3,0	2,0	

1.2. Generic MA 5P Matrix

EUROPEAN BACCALAUREATE - Generic MA 5P Matrix

Element of Examination	Question	Learning Objective (specific syllabus reference(s))	Paper-specific Marking Scheme				
			Knowledge and Comprehension	Methods	Problem Solvng	Interpretation and Linking	Σ
Part A - Non Calculator							
Analysis	A1						0,0
Geometry	A2						0,0
Probability	A3						0,0
Sequences	A4						0,0
Complex Numbers	A5						0,0
Analysis or Geom or Prob	A6						0,0
Analysis or Geom or Prob	A7						0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	7,5	12,0	9,0	1,5	30,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	1,0	2,0	2,0	1,0	

Part B - Calculator							
B1							0,0
Analysis							0,0
							0,0
Minimum 4 sub-questions							0,0
							0,0
							0,0
							0,0
Maximum 8 sub-questions							0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	
B2							0,0
Geometry							0,0
							0,0
Minimum 4 sub-questions							0,0
							0,0
							0,0
							0,0
Maximum 8 sub-questions							0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

B3							0,0
Probability							0,0
							0,0
Minimum 4 sub-questions							0,0
							0,0
							0,0
							0,0
Maximum 8 sub-questions							0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	
B4 or B4 and B5	B4a						0,0
Sequences	B4b						0,0
and or	B4c						0,0
Complex Numbers	B5a						0,0
	B5b						0,0
	B5c						0,0
							0,0
							0,0
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	2,5	4,0	3,0	0,5	10,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

Total Part B - Calculator						
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	17,5	28,0	21,0	3,5	**70,0**
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

Total A and B						
		S	0,0	0,0	0,0	0,0	0,0
		%	0,0	0,0	0,0	0,0	
		Guideline:	25,0	40,0	30,0	5,0	**100,0**
		Tolerance (Points):	3,0	5,0	4,0	2,0	

2. Paper-specific Matrices

These matrices use the 2019 Baccalaureate papers to illustrate the procedures laid out in the Matrix Manual Mathematics, which accompanies this document.

2.1. MA 3P (Based on the Baccalaureate of 11th June 2019)

EUROPEAN BACCALAUREATE - Paper specific Ma 3P Matrix (11th JUNE 2019)

Element of Examination	Question	Learning Objective (specific syllabus reference(s))	Paper-specific Marking Scheme				
			Knowledge and Comprehension	Methods	Problem Solvng	Interpretation and Linking	Σ
Non Calculator							
Analysis	A1	Solve an exponential equation	3,0	2,0			5,0
Analysis	A2	Determine an equation of a tangent	1,0	2,0	2,0		5,0
Analysis	A3	Sketch a possible graph	2,0	2,0	1,0		5,0
Analysis	A4	Determine a primitive function	1,0	3,0	1,0		5,0
Analysis	A5	Calculate an area	2,0	2,0	1,0		5,0
Probability	A6	Calculate a probability	2,0	2,0	1,0		5,0
Probability	A7	Calculate a binomial probability	2,0	3,0			5,0
Statistics	A8	Determine the median, quartiles and represent on a boxplot	2,0	3,0			5,0
		S	**15,0**	**19,0**	**6,0**	**0,0**	**40,0**
		%	**37,5**	**47,5**	**15,0**	**0,0**	
		Guideline:	12,0	18,0	8,0	2,0	40,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	
Calculator							
B1	B1a	Sketch Graphs-Determine Points of intersection	2,0	2,0			4,0
Analysis	B1b	Calculate area of a region bounded by	1,0	1,0			2,0
Exactly 3 sub questions	B1c	Use parallelism line/tangent line	0,0	2,0	2,0		4,0
		S	**3,0**	**5,0**	**2,0**	**0,0**	**10,0**
		%	**30,0**	**50,0**	**20,0**	**0,0**	
		Guideline:	3,0	4,5	2,0	0,5	10,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	
B2	B2a	Exponential function-calculate values	1,0	1,0			2,0
Analysis	B2b	Draw the graph	1,0	2,0			3,0
	B2c	Interpret limit of a function			1,0	2,0	3,0
Minimum 4 sub questions	B2d	Solve an exponential equation	2,0	1,0			3,0
Maximum 5 sub questions	(B2e)	Interpret maximum growth rate	1,0	2,0	1,0		4,0
		S	**5,0**	**6,0**	**2,0**	**2,0**	**15,0**
		%	**33,3**	**40,0**	**13,3**	**13,3**	
		Guideline:	4,5	6,8	3,0	0,8	15,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	

B3	B3a	Calculate a conditinal probability/Normal		2,0	1,0		3,0
Probability	B3b	Calculate a conditional probability		1,0	2,0		3,0
	B3c	idem			1,0	2,0	3,0
Minimum 4 sub-questions	B3d	Calculate a probability/Binomial distribution	2,0	1,0			3,0
Maximum 5 sub-questions	(B3e)	idem	1,0	2,0			3,0
		S	**3,0**	**6,0**	**4,0**	**2,0**	**15,0**
		%	**20,0**	**40,0**	**26,7**	**13,3**	
		Guideline:	4,5	6,8	3,0	0,8	15,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	
B4 (and B5)	B4a	Expon. model: draw a scatter plot and the	2,0	3,0			5,0
Statistics	B4b	Estimate the value	1,0	1,0			2,0
	B4c	Solve an exponential equation		2,0	1,0		3,0
If there is no B5, B4 must have...	B4d or B5a	Determine equation of exponential regression		4,0			4,0
Minimum 5 sub-questions	B4e or B5b	Analyse annual percentage growth rate	1,0	1,0	1,0		3,0
Maximum 6 sub-questions	B4f or B5c	Estimate and comment on models			2,0	1,0	3,0
		S	**4,0**	**11,0**	**4,0**	**1,0**	**20,0**
		%	**20,0**	**55,0**	**20,0**	**5,0**	
		Guideline:	6,0	9,0	4,0	1,0	20,0
		%	30,0	45,0	20,0	5,0	
		Tolerance (Points):	3,0	4,0	2,0	2,0	

Total Calculator						
	S	**15,0**	**28,0**	**12,0**	**5,0**	**60,0**
	%	**25,0**	**46,7**	**20,0**	**8,3**	
	Guideline:	18,0	27,0	12,0	3,0	60,0
	%	30,0	45,0	20,0	5,0	
	Tolerance (Points):	3,0	4,0	2,0	2,0	

Total						
	S	**30,0**	**47,0**	**18,0**	**5,0**	**100,0**
	%	**30,0**	**47,0**	**18,0**	**5,0**	
	Guideline:	30,0	45,0	20,0	5,0	100,0
	%	30,0	45,0	20,0	5,0	
	Tolerance (Points):	4,0	5,0	3,0	2,0	

The fields marked in yellow can be filled in, all others are protected.		
The Sums are	green:	ok
	orange:	within tolerance
	red:	not allowed

For each individual question in the calculator paper (B1, B2 etc.), there is more flexibility in the spread of the marks but the overall weighting of the marks for the whole calculator paper must be respected.

2.2. MA 5P (Based on the Baccalaureate of 11th June 2019)

EUROPEAN BACCALAUREATE - Paper specific MA 5P Matrix (19th June 2019)

Element of Examination	Question	Learning Objective (specific syllabus reference(s))	Paper-specific Marking Scheme				
			Knowledge and Comprehension	Methods	Problem Solvng	Interpretation and Linking	Σ
Non Calculator							
Analysis	A1	understand the concept of a primitive. P(x)/Q(x) where P(x) and Q(x) are polynomials of degree two or less	1,0	1,0	2,0		4,0
Geometry	A2	determine the relative position line/sphere	1,0	3,0	0,0		4,0
Probability	A3	tree diagrams of conditional events (sampling without replacement)	1,0	2,0	2,0		5,0
Sequences	A4	Calculate a limit of sequence from the basic set of recurrence relations for simple cases	1,0	2,0	1,0		4,0
Complex Numbers	A5	determine the magnitude and argument of the product and quotient of two complex numbers	2,0	1,0	1,0		4,0
Analysis or Geom or Prob	A6	examine the following characteristics for all the basic functions given above: tangent at a point		1,0	2,0	1,0	4,0
Analysis or Geom or Prob	A7	determine the relative position of point/line		2,0	2,0	1,0	5,0
Total Non Calculator							
		S	6,0	12,0	10,0	2,0	30,0
		%	20,0	40,0	33,3	6,7	
		Guideline:	7,5	12,0	9,0	1,5	30,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	1,0	2,0	2,0	1,0	
Calculator							
B1	a	examine the following characteristics for all the basic functions given above: asymptotes	1,0	1,0			2,0
Analysis	b	examine the following characteristics for all the basic functions given above: derivatives	2,0	1,0			3,0
	c	examine the following characteristics for all the basic functions given above: inflexion points	1,0	2,0			3,0
Minimum 4 sub questions	d	integration to calculate area in a plane	1,0	2,0			3,0
	e	examine the following characteristics for all the basic functions given above: continuity	2,0	1,0			3,0
	f	examine the following characteristics for all the basic functions given above: differentiability		2,0	1,0		3,0
	g	integration to calculate area in a plane		1,0	2,0		3,0
Maximum 8 sub questions							0,0
		S	7,0	10,0	3,0	0,0	20,0
		%	35,0	50,0	15,0	0,0	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	
B2	a	find parametric and cartesian equations of a plane (S6)	1,0	2,0			3,0
Geometry	b	calculate the acute angle if it exists made by: line/plane		3,0			3,0
	c	the analytical expression of the scalar product (dot product) of two vectors, the length of a vector		1,0	2,0		3,0
Minimum 4 sub questions	d	using c	1,0	1,0			2,0
	e	determine orthogonal projections			2,0	1,0	3,0
	f	determine the relative position: line/plane		1,0	2,0		3,0
	g	determine the relative position: point/sphere		1,0	2,0		3,0
Maximum 8 sub questions							0,0
		S	2,0	9,0	8,0	1,0	20,0
		%	10,0	45,0	40,0	5,0	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

B3	a	tree diagrams of independent events (sampling with replacement) (S6)	1,0	2,0			3,0
Probability	b	Bayes' Theorem	1,0	2,0			3,0
	c	calculate probabilities for a binomially distributed random variable	2,0	1,0			3,0
Minimum 4 sub questions	d	find cumulative probabilities		1,0	2,0		3,0
	e	investigate data given in a table or a diagram to determine a corresponding normal distribution	1,0				1,0
	f	cumulative distribution function for a continuous random variable and its relation to integral calculus	1,0	1,0			2,0
	g	mean (expected value), variance and standard deviation of a continuous random variable	2,0	1,0			3,0
Maximum 8 sub questions	h	use the normal distribution with probability of independent events		1,0	1,0		2,0
		S	**8,0**	**9,0**	**3,0**	**0,0**	**20,0**
		%	**40,0**	**45,0**	**15,0**	**0,0**	
		Guideline:	5,0	8,0	6,0	1,0	20,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	
B4 or B4 and B5	B4a	calculate terms of a sequence using the types of definitions above.		1,0			1,0
Sequences	B4b	solve problems involving the properties of arithmetic and geometric sequences			1,0	1,0	2,0
and/or	B4c	given the first terms of a sequence find the explicit nth term and or the recurrence relation where appropriate			1,0	1,0	2,0
Complex Numbers	B5a	represent a complex number geometrically	1,0				1,0
	B5b	determine the magnitude and argument of the product and quotient of two complex numbers		2,0			2,0
	B5c	determine the magnitude and argument of the product and quotient of two complex numbers		2,0			2,0
							0,0
							0,0
		S	**1,0**	**5,0**	**2,0**	**2,0**	**10,0**
		%	**10,0**	**50,0**	**20,0**	**20,0**	
		Guideline:	2,5	4,0	3,0	0,5	10,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

Total Calculator						
		S	**18,0**	**33,0**	**16,0**	**3,0**	**70,0**
		%	**25,7**	**47,1**	**22,9**	**4,3**	
		Guideline:	17,5	28,0	21,0	3,5	**70,0**
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	4,0	3,0	1,0	

Total						
		S	**24,0**	**45,0**	**26,0**	**5,0**	**100,0**
		%	**24,0**	**45,0**	**26,0**	**5,0**	
		Guideline:	25,0	40,0	30,0	5,0	**100,0**
		Tolerance (Points):	3,0	5,0	4,0	2,0	

3.Sample Bac written examination

For reference the MA-3P 11th June 2019 paper is included here. The same approach can be applied to past BAC Exams for 3P and 5P.

EUROPEAN BACCALAUREATE 2019

MATHEMATICS 3 PERIODS
PART A

DATE : 11th June 2019 Afternoon

DURATION OF THE EXAMINATION:

1 hour (60 minutes)

AUTHORIZED MATERIAL:

Examination without technological tool

Pencil for the graphs

SPECIFIC INSTRUCTIONS:

- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.

- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.

- When the answer provided is not the correct one, some marks can be awarded if it is evident that an appropriate method and/or a correct approach has been used.

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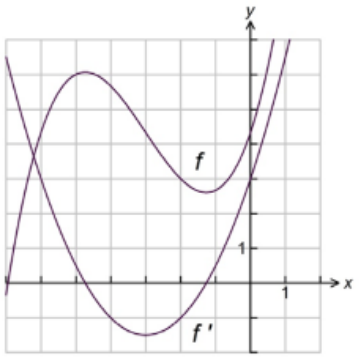
EUROPEAN BACCALAUREATE 2019: MATHEMATICS 3 PERIODS

PART A

Page 1/2 | **Marks**

1) Solve the equation $e^{4x-1} = 1$. — 5 marks

2) The diagram below shows the graph of a function f and the graph of the derivative f' of f.



Determine an equation of the tangent to the graph of f at the point where $x = -2$. — 5 marks

3) The table below gives information concerning the function f and its derivative f'.

x	-4	-3	-2	-1	0
$f(x)$	0	4	2	0	4
$f'(x)$	+	0	−	0	+

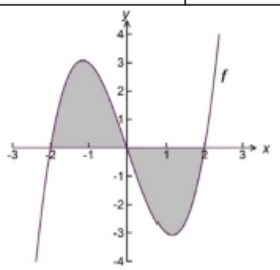
Sketch a possible graph of this function f. — 5 marks

4) Consider the function f defined by

$$f(x) = 2x + 3 + \frac{1}{x+3}, \quad x > -3.$$

Determine the primitive F of f given that $F(-2) = 2$. — 5 marks

EUROPEAN BACCALAUREATE 2019: MATHEMATICS 3 PERIODS

PART A		
	Page 2/2	**Marks**
5) The diagram shows the graph of the function f defined by $f(x) = x^3 - 4x$. Calculate the area of the shaded region. 		5 marks
6) In a class of 21 students 12 students study Biology, 14 students study Music and 2 students study neither Biology nor Music. Calculate the probability that a student selected at random from this class studies both Biology and Music.		5 marks
7) In an experiment, slices of toasted bread are buttered on one side. The probability that a slice lands butter side down if you drop it is $\frac{3}{5}$. 3 slices are dropped. Calculate the probability that exactly 2 of these slices land butter side down.		5 marks
8) 10 students score the following marks in a test: 10 2 5 7 8 5 6 7 8 4 . Determine the median, the lower and upper quartiles, and represent the data on a boxplot.		5 marks

EUROPEAN BACCALAUREATE 2019

MATHEMATICS 3 PERIODS
PART B

DATE: 11th June 2019 Morning

DURATION OF THE EXAMINATION:

2 hours (120 minutes)

AUTHORIZED MATERIAL:

Examination with technological tool:

Calculator TI-Nspire in "Press-to-test" mode

Pencil for the graphs

SPECIFIC INSTRUCTIONS:

- Use a different page for each question.
- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.
- Some of the questions can be answered only with the help of the calculator. The wording of these questions makes this clear. All other questions can be solved with or without the use of the calculator.

PART B		
QUESTION B1 ANALYSIS	**Page 1/1**	**Marks**

Consider the functions f and g defined by

$$f(x) = -x^2 - 2x + 5 \quad \text{and} \quad g(x) = x + 1.$$

a) Sketch the graphs of f and g on the same diagram.
Determine the coordinates of their points of intersection. — 4 marks

b) The area A of the region bounded by the graphs of two functions f and g between the x-values a and b is given by:

$$A = \int_a^b |f(x) - g(x)| dx \ .$$

Calculate the area of the region bounded by the graphs of f and g between the x-values -4 and 1. — 2 marks

c) Determine the x-coordinate of the point on the graph of f where the tangent line is parallel to the graph of g. — 4 marks

PART B

QUESTION B2 ANALYSIS	Page 1/1	Marks

Use your calculator in a), b), d), and e).

In an experiment the steeping time for green tea leaves is studied.
Hot water is poured over the tea leaves.
The theine in the tea leaves is then dissolved in the hot water.
The theine content in the hot tea as a function of time is modelled by the function f defined by

$$f(x) = 48 \cdot (1 - e^{-0.6x}) ,$$

where $f(x)$ is the theine content in the hot tea, measured in mg per gram of tea, and x is the time in minutes after the hot water was poured over the tea leaves.

a) Calculate the theine content after 1 minute and after 6 minutes. — 2 marks

b) Draw the graph of f for the first 10 minutes. — 3 marks

c) Interpret the factor 48 in the above equation. — 3 marks

d) The tea is ready to drink when the theine content reaches 33.6 mg/g.
Determine at what time the tea will be ready to drink. — 3 marks

e) The tea also contains tannin. The tannin content in the hot tea as a function of time is modelled by the function

$$g(x) = \frac{37}{1 + e^{-3x+6}} ,$$

where $g(x)$ is the tannin content in the hot tea, measured in mg per gram of tea, and x is the time in minutes after the hot water was poured over the tea leaves.
The taste of tea is best when the tannin content growth rate $g'(x)$ is at its maximum.
Determine at what time the taste of the tea will be best. — 4 marks

PART B		
QUESTION B3 PROBABILITY	**Page 1/1**	**Marks**

Use your calculator in all questions.

A factory has two machines, one fills pineapple juice into cans and the other fills iced tea into cans.

The requirement is that each can contains 33 centilitres (cL). Cans which contain less than 31.5 cL or more than 34 cL are classified as incorrectly filled.

a) The volume of pineapple juice filled into each can follows a normal distribution with mean $\mu = 32.5$ cL and standard deviation $\sigma = 0.5$ cL.

A can of pineapple juice is selected at random.

Show that the probability that this can is incorrectly filled is 0.0241. — 3 marks

40 % of all cans filled at the factory are cans of iced tea.
3.25 % of the cans of iced tea are classified as incorrectly filled.

b) From the factory, a can is chosen at random.

Show that the probability that this can is classified as incorrectly filled is 0.0275. — 3 marks

c) Given that a randomly selected can is incorrectly filled, calculate the probability that it contains pineapple juice. — 3 marks

The pineapple cans are packaged in packs of 6.

d) Calculate the probability that there is exactly one incorrectly filled can in a randomly selected pineapple juice six-pack. — 3 marks

e) Calculate the probability that there is more than one incorrectly filled can in a randomly selected pineapple juice six-pack. — 3 marks

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PART B

QUESTION B4 STATISTICS | **Page 1/1** | **Marks**

Use your calculator in a), b), c), d) and f).

The table below shows the global production of plastic from 2010 to 2013.

Year		2010	2011	2012	2013
Time in years after 2010	x	0	1	2	3
Plastic production (million tonnes)	y	313	325	338	352

Source: https://www.theatlas.com/charts/BkAVFsjrb

The function f defined by

$$f(x) = e^{5.745+0.040x}$$

is an exponential model based on the data given.
$f(x)$ is an estimate of the plastic production in million tonnes at time x in years after 2010.

a) On the same diagram, draw a scatter plot representing the data from the table, and draw the graph of the function f. — 5 marks

b) Using the function f, estimate the plastic production for 2015. — 2 marks

c) Using the function f, estimate in which year the plastic production will, for the first time, exceed 450 million tonnes. — 3 marks

d) Determine an equation in the form $y = a \cdot b^x$ of the exponential regression of y on x using the data given.
Give the number b correct to four decimal places. — 4 marks

For e) and f) use the exponential regression model g, where

$$g(x) = 313 \cdot 1.040^x.$$

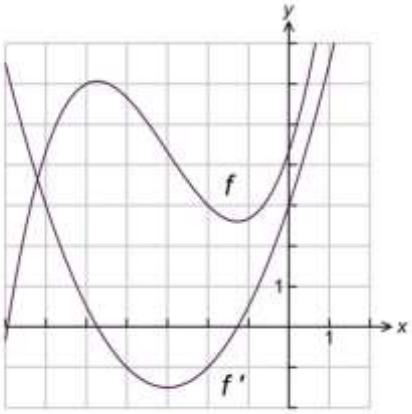
e) What is the annual percentage growth rate in the model g? — 3 marks

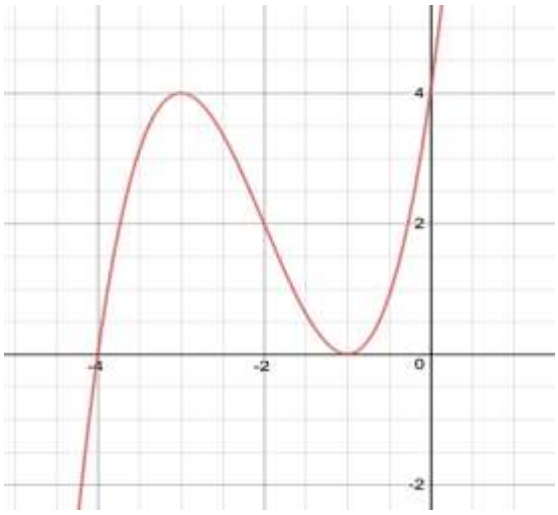
f) Using each of the two models, estimate the plastic production in 2020.
Comment on your results. — 3 marks

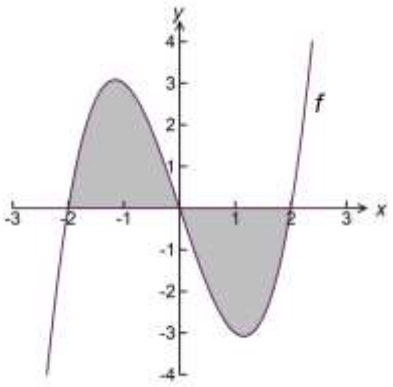
5/5

4. Marking scheme

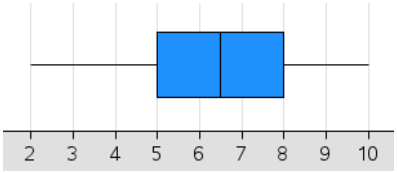
Below is an example of a marking scheme linked to the maths matrix. The 3p paper has been used to illustrate the formatting. For brevity, the 5p mark scheme has been omitted as, given the common approach being adopted for the 3p and 5p course, it would follow the same structure as the example below.

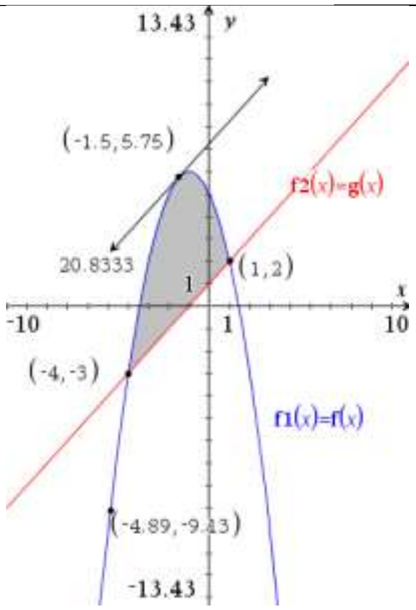
Questions Part A (3P) - 2019						
1.Knowledge and Comprehension 2.Methods 3. Problem Solving 4. Interpretation and Linking	1.	2.	3.	4.	Σ	**Learning Objectives / Tasks**
A1						Analysis
Solve the equation $e^{4x-1}=1$.					5	**Exponen-tial equation**
$4x-1=0$ solution $x=\frac{1}{4}$	3	2				**S7: Define** the exponential function **S2**: **Solve** an equation
A2						Analysis
The diagram below shows the graph of a function *f* and the graph of the derivative f' of *f*.  Determine an equation of the tangent to the graph of *f* at the where $x=-2$.					5	**Graph of a function and its derivative**
The tangent line to the graph of *f* in its point having *x*-coordinate equal to -2 has equation						S6:

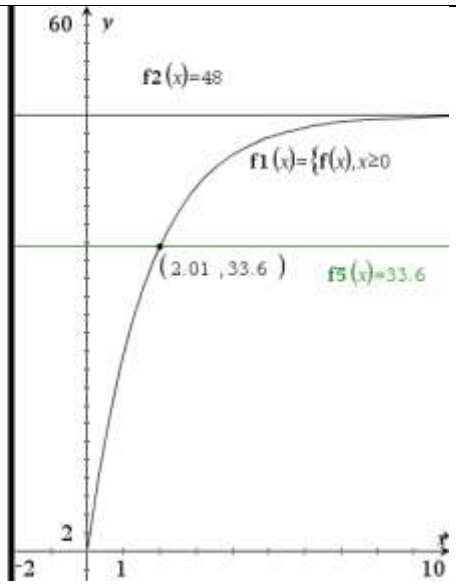
$y - f(-2) = f'(-2)(x + 2)$. $f(-2) = 3$ $f'(-2) = -1$ $y - 3 = -(x + 2)$ i.e. $y = -x + 1$ OR: $f'(-2) = -1$ $y = -1x + c$ $f(-2) = 3$ $y = -x + 1$ *For a solution using the graph of f only, award maximum 3 p* solution $x = \frac{1}{4}$	1 1	2 2	2 2			**Know** the formulae for a tangent to the graph **Apply/ use** the graphs **Solve (Calculate and reduce)** **Use** the graph **Apply** a formula of tangent **Solve**
A3						Analysis
The table below gives information concerning the function *f* and its derivative f'. | x	-4	-3	-2	-1	0 | |---|---|---|---|---|---| | $f(x)$ | 0 | 4 | 2 | 0 | 4 | | $f'(x)$ | $+$ | 0 | $-$ | 0 | $+$ | Sketch a possible graph of this function *f*.					5	**Show understanding** of a function and its derivative
For instance:  	1	3	1			**S6: Show understanding** of … **Sketch** a possible graph

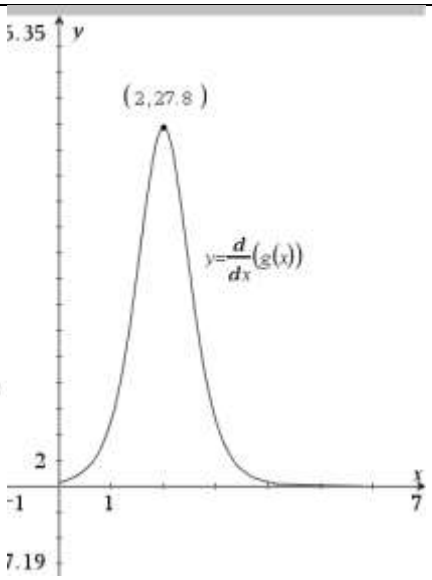
A4						Analysis
Consider the function *f* defined by $f(x) = 2x + 3 + \frac{1}{x+3}, \qquad x > -3$. Determine the primitive *F* of *f* given that $F(-2) = 2$.					5	
For $x > -3$ we have: $\int f(x)\,dx = x^2 + 3x + \ln(x+3) + k$ Using $F(-2) = 2$ $(-2)^2 + 3 \cdot (-2) + \ln 1 + k = 2$ $4 - 6 + k = 2$, from which we get $k = 4$. The function *F* may thus be expressed by $F(x) = x^2 + 3x + \ln(x+3) + 4$	2	2	1			**S7:** **Determine** a primitive **Apply** a condition **Determine** the primitive given that …
A5						Analysis
The diagram shows the graph of the function *f* defined by $f(x) = x^3 - 4x$.  Calculate the area of the shaded region.					5	**Area under the graph**
The graph intersects the *x*-axis at $x = -2,\ x = 0,\ x = 2$. The area *A* of the shaded region is given by $A = \int_{-2}^{2} \lvert f(x) \rvert\,dx = 2\int_{-2}^{0} f(x)dx = 2\int_{0}^{2} (-f(x))\,dx$ since the graph is symmetrical with respect to the origin. $A = 2\int_{-2}^{0} (x^3 - 4x)dx = 2 \cdot \left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} = 8$	2	2	1			**S7:** **Define** the area under the graph **Recognize** **Determine** the integral **Interpret**

A6						Probability
In a class of 21 students 12 study Biology, 14 study Music and 2 study neither Biology nor Music. Calculate the probability that a student selected at random from this class studies both Biology and Music.					5	**Elementary probability**
There are 19 students who take biology and/or music. Therefore, there are $14 + 12 - 19 = 7$ who take both biology and music. $P(student\ takes\ biology\ and\ music) = \frac{7}{21} = \frac{1}{3}$.	2	2	1			S7: Probability **Analyse** and **explain** **Calculate** a probability
A7						Probability
In an experiment, slices of toasted bread are buttered on one side. The probability that a slice lands butter side down if you drop it is $\frac{3}{5}$. 3 slices are dropped. Calculate the probability that exactly 2 of these slices land butter side down.					5	**Binomial distribution**
$P(exactly\ two\ butter\ side\ down) = 3 \cdot \left(\frac{3}{5}\right)^2 \cdot (1 - \frac{3}{5}) = \frac{54}{125}$	2	3				**Recognize** binomial parameters
A8						Statistics
10 students score the following marks in a test: 10 2 5 7 8 5 6 7 8 4 . Determine the median, the lower and upper quartiles, and represent the data on a boxplot.					5	**Elementary statistics**

Rearranging data: 2 4 5 5 6 7 7 8 8 10 Median is 6.5, lower quartile is 5, upper quartile is 8  Boxplot	2	3				S7: **Determine** the median, the quartiles and **represent** on a boxplot

Questions Part B (3P) - 2019						
1. Knowledge and Comprehension 2. Methods 3. Problem Solving 4. Interpretation and Linking	1.	2.	3.	4.	Σ	**Learning Objectives/ Tasks**
B1						**Analysis**
Consider the functions *f* and *g* defined by $f(x) = -x^2 - 2x + 5$ and $g(x) = x + 1$. a) Sketch the graphs of *f* and *g* on the same diagram. Determine the coordinates of their points of intersection. — 4 marks b) The area *A* of the region bounded by the graphs of two functions *f* and *g* between the *x*-values *a* and *b* is given by: $A = \int_a^b \lvert f(x) - g(x) \rvert dx$. Calculate the area of the region bounded by the graphs of *f* and *g* between the *x*-values –4 and 1. — 2 marks c) Determine the *x*-coordinate of the point on the graph of *f* where the tangent line is parallel to the graph of *g*. — 4 marks					10	**S6/S7:** *Revisiting linear and quadratic models* *Area of the region bounded by two graphs and between x-values*
$f(x):=-x^2-2\cdot x+5$ ▸ *Fertig* $g(x):=x+1$ ▸ *Fertig* a) See → Found using the graph: Intersection points (−4,−3) and (1,2) . The intersection points can also be found by solving the equation $f(x)=g(x)$: solve$(f(x)=g(x),x)$ ▸ $x=-4$ or $x=1$ b) The area equals: $\int_{-4}^{1}(f(x)-g(x))dx = \frac{125}{6} \approx 20.8333$. The area can also be determined graphically. 	2 1	2 1				**Sketch** the graphs **Determine** the coordinates of points of intersection **Calculate** the area of the region bounded by two graphs and between two x-values
c) The derivative *fp* of *f* : $fp(x):=\frac{d}{dx}(f(x))$ ▸ *Fertig* solve$(fp(x)=1,x)$ ▸ $x=\frac{-3}{2}$ **The x−coordinate is $\frac{-3}{2}$.** The tangent line at this point is drawn on the graph (not required by the students)		2	2			**Determine** the x-coord. **Explore** the relationship between graphs/derivat. **Characterize** the parallelism of two lines

B2						Analysis
Use your calculator in a), b), d), and e). In an experiment the steeping time for green tea leaves is studied. Hot water is poured over the tea leaves. The theine in the tea leaves is then dissolved in the hot water. The theine content in the hot tea as a function of time is modelled by the function f defined by $f(x) = 48 \cdot (1 - e^{-0.6x})$, where $f(x)$ is the theine content in the hot tea, measured in mg per gram of tea, and x is the time in minutes after the hot water was poured over the tea leaves. a) Calculate the theine content after 1 minute and after 6 minutes. b) Draw the graph of f for the first 10 minutes. c) Interpret the factor 48 in the above equation. d) The tea is ready to drink when the theine content reaches 33.6 mg/g. Determine at what time the tea will be ready to drink. e) The tea also contains tannin. The tannin content in the hot tea as a function of time is modelled by the function $g(x) = \frac{37}{1 + e^{-3x+6}}$, where $g(x)$ is the tannin content in the hot tea, measured in mg per gram of tea, and x is the time in minutes after the hot water was poured over the tea leaves. The taste of tea is best when the tannin content growth rate $g'(x)$ is at its maximum. Determine at what time the taste of the tea will be best.					15	**S7:** *Exponential functions*
$f(x) := 48 \cdot \left(1 - e^{-0.6 \cdot x}\right)$ • *Fertig* a) $f(1) = 21.657$ $f(6) = 46.6885$ **Concentration after 1 minute: 21.6 mg/g** **Concentration after 6 minutes: 46.7 mg/g** b) **see graph →** c) $\lim_{x \to \infty} (f(x)) = 48$ **The upper limit of the theine concentration is 48 mg/g and the tea is ready to drink** **See graph →** d) $\text{solve}(f(x) = 33.6, x)$ • $x = 2.00662$ **After 2.0 minutes the concentration is 33.6 mg/g** **Can also be found by using the intersection command.**		1 1 1 2 2 1	1 1 2 1	1 2		**Calculate** the y-value **Draw** the graph **Calculate** a limit/**Interpret** a factor **Solve** an equation

e) $g(x):=\frac{37}{1+e^{-3\cdot x+6}}$ • *Fertig* $gp(x):=\frac{d}{dx}(g(x))$ • *Fertig* $fMax(gp(x),x)$ • $x=2$ Can also be found on the graph. Can also be found by solving the equation: $solve\left(\frac{d^2}{dx^2}(g(x))=0,x\right)$ • $x=2$ **Tea is best after 2 minutes.** 	1	2	1			**Calculate** the derivative **Explore** the derivative and how it may vary **Solve** an equation

B3						Probability
Use your calculator in all questions. A factory has two machines, one fills pineapple juice into cans and the other fills iced tea into cans. The requirement is that each can contains 33 centilitres (cL). Cans which contain less than 31.5 cL or more than 34 cL are classified as incorrectly filled. a) The volume of pineapple juice filled into each can follows a normal distribution with mean $\mu = 32.5$ cL and standard deviation $\sigma = 0.5$ cL. A can of pineapple juice is selected at random. Show that the probability that this can is incorrectly filled is 0.0241. 40 % of all cans filled at the factory are cans of iced tea. 3.25 % of the cans of iced tea are classified as incorrectly filled. b) From the factory, a can is chosen at random. Show that the probability that this can is classified as incorrectly filled is 0.0275. c) Given that a randomly selected can is incorrectly filled, calculate the probability that it contains pineapple juice. The pineapple cans are packaged in packs of 6. d) Calculate the probability that there is exactly one incorrectly filled can in a randomly selected pineapple juice six-pack. e) Calculate the probability that there is more than one incorrectly filled can in a randomly selected pineapple juice six-pack.					15	**S6:***General probability rules, Dependent events, Conditional probabilities* **S7:***Normal distribution*
a) P(incorrectly filled from A) = 1−normCdf(31.5,34,32.5,0.5) = **0.0241** or normCdf(-∞,31.5,32.5,0.5)+normCdf(34,∞,32.5,0.5) = 0.0241 b) P(incorrectly filled) = P(incorrectly filled|A)· $P(A)$+P(incorrectly filled|B)· $P(B)$ = 0.0241· 0.6+0.0325· 0.4 = **0.02746** **i.e. 2.75 % of all cans are classified as incorrectly filled** c) P(pineapple | incorrectly filled)=$\frac{P(\text{pineapple} \cap \text{incorrectly filled})}{P(\text{incorrectly filled})}$ = $\frac{P(\text{incorrectly filled} \mid \text{pineapple}) \cdot P(\text{pineapple})}{P(\text{incorrectly filled})}$ = $\frac{0.0241 \cdot 0.6}{0.02746}$ = **0.526584** or by using the rounded off result from b): $\frac{0.0241 \cdot 0.6}{0.0275}$ = **0.525818** d) P(exactly 1 incorrectly filled can in the 6−pack) = binomPdf(6,0.0241,1) = **0.127996** ≈ **0.128** e) P(more than 1 incorrectly filled can in the 6−pack) = binomCdf(6,0.0241,2,6) = **0.008167** ≈ **0.0082** .	2 1	2 1 1	1 2 1 2	2		**Calculate** a probability (normal distribution **Know** the rules for a conditional probability **Investigate, connect** and **apply** **Calculate** probabilities for a random variable with a binomial distribution Id.

B4							Statistics
Use your calculator in a), b), c), d) and f). The table below shows the global production of plastic from 2010 to 2013.						20	**S7:** *Visualization, Correlation, Regression*

Year		2010	2011	2012	2013
Time in years after 2010	x	0	1	2	3
Plastic production (million tonnes)	y	313	325	338	352

Source: https://www.theatlas.com/charts/BkAVFsjrb

The function f defined by

$$f(x) = e^{5.745+0.040x}$$

is an exponential model based on the data given.
$f(x)$ is an estimate of the plastic production in million tonnes at time x in years after 2010.

a) On the same diagram, draw a scatter plot representing the data from the table, and draw the graph of the function f.

b) Using the function f, estimate the plastic production for 2015.

c) Using the function f, estimate in which year the plastic production will, for the first time, exceed 450 million tonnes.

d) Determine an equation in the form $y = a \cdot b^x$ of the exponential regression of y on x using the data given.
Give the number b correct to four decimal places.

For e) and f) use the exponential regression model g, where

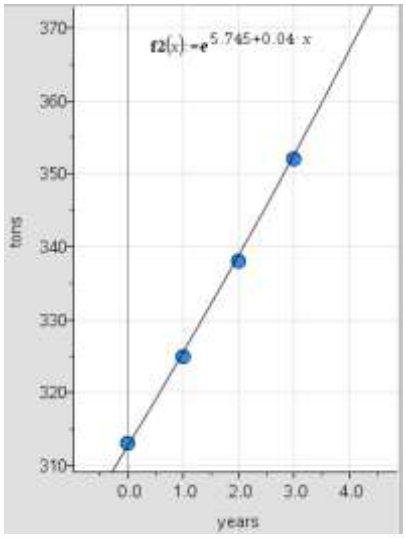
$$g(x) = 313 \cdot 1.040^x.$$

e) What is the annual percentage growth rate in the model g?

f) Using each of the two models, estimate the plastic production in 2020.
Comment on your results.

a) $\mathbf{f}(x) := e^{5.745+0.04\cdot x}$ ▸ *Fertig*

Scatter plot and Graph →



b) $\mathbf{f}(5) = 382$. **382 million tons in 2015**

c) solve$(\mathbf{f}(x)=450, x)$ ▸ $x=9.10619$

$x=10$ (2020): $\mathbf{f}(10) = 466.38$

Exceeds 450 million tons for the first time in 2020.

d) See spreadsheet next page. Result:

$\mathbf{f1}(x) = 312.765\cdot(1.03993)^x$.

Hence ***b*=1.0399**

	ye...	tons		=ExpReg(
1	0	313	Title	Régress...
2	1	325	RegEqn	a*b^x
3	2	338	a	312.765
4	3	352	b	1.03993
5			r^2	0.999713
6			r	0.999857
7			Resid	{0.23545..
8			ResidTra..	{7.52529..
9				
10				
11				
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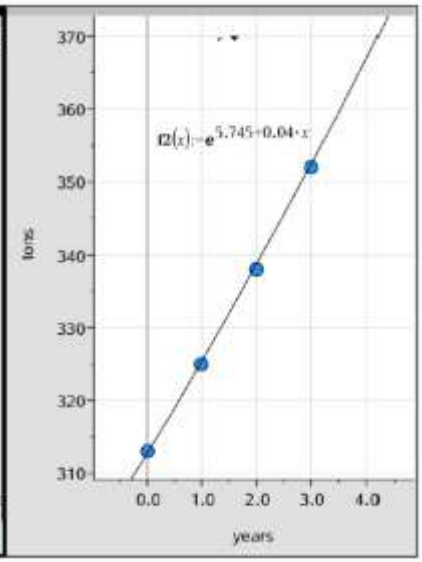

e) $\mathbf{g}(x) := 313\cdot(1.04)^x$ ▸ *Fertig*

Annual percentage growth rate

$(1.04-1)\cdot 100$ ▸ 4. % = **4.0 % per year.**

f) Entering the expression into the calculator it is automatically rewritten

$e^{5.745+0.04\cdot x} = 312.624\cdot(1.04081)^x$ or $\mathbf{f}(x) = 312.624\cdot(1.04081)^x$

or with powers: $e^{5.745} = 312.624$ and $e^{0.04\cdot x} = (1.04081)^x$

Rounded off the two models *f* and *g* are the same.

2	3				**Draw** a scatter plot/graph
1	1				**Estimate** a value
	2	1			**Apply** and **estimate**
	4				**Determine** an equation of the exponential regression
1	1	1			**Use** a regression model **Interpret**
		2	1		**Analyse** and **comment** on results